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Effects of Configuration Mixing on the 3⁻ State of Pb^{208} and the (p,p') Reaction in the Distorted-Wave Approximation*

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In this paper it is shown that the experimental mean life of the first excited 3^- state of Pb^{208} can be approximately accounted for if configuration mixing is introduced instead of considering this state as a pure single-particle shell-model state. For this purpose an octupole-octupole force is assumed (an extension of the quadrupole-quadrupole force of Elliott). Further, the direct interaction theory of inelastic scattering in the distorted-wave Born approximation is used to calculate the cross section of 23-MeV protons for excitation of the 0.57-MeV and 0.90-MeV levels of Pb^{207} [corresponding to the transitions $(p_{1/2})^{-1}\rightarrow(f_{5/2})^{-1}$ and $(p_{1/2})^{-1}\rightarrow(p_{3/2})^{-1}$, respectively] and for excitation of the anomalous peak at 2.6 MeV (3^- state of Pb^{208}) using the configuration-mixed nuclear wave function. It is suggested that the anomalous peak at 2.6 MeV for Pb^{206} and Pb^{207} is also due to superposition of many single-particle transitions as in Pb^{208}. As a result of the configuration mixing, reasonable agreement is obtained between the experimental and theoretical cross sections for the above-mentioned cases. Finally, various suggestions which can lead to better understanding of this process are made.

I. INTRODUCTION

In a previous paper,[1] we analyzed the experimental data on the inelastic scattering of 23-MeV protons reported by Cohen, Mosko, and Rubin.[2–4] Our basic idea was to consider the inelastic scattering as being well described by the theory of direct interaction in the plane wave approximation.[5,6] We applied this idea to calculate the inelastic scattering cross section of nuclei for which the structure of the lower-lying levels is fairly well known. It is found that the above theory could well explain the experimental results on the relative magnitude of the excitation of the various states. In particular, the anomalous peak at $Q=-2.6$ MeV in the Pb isotopes was considered as due to the proton transition, $(d_{3/2})\rightarrow(h_{9/2})$, which is commonly considered to explain the 3^- state of Pb^{208}. The mean life of this state has been measured by Crannel *et al.*[7] and reported to be $4(\pm2)\times10^{-11}$ sec; on the other hand, the theoretical value of this life calculated using the Weisskopf single-particle formula[8] is approximately 7×10^{-10} sec, if $1.18A^{1/3}$F is used for the radius of the charge distribution. This result means that the enhancement of the $E3$ transition probability over the single-particle value is larger by a factor of ten to thirty-five and this fact may support the view that the first excited 3^- state of Pb^{208} is an octupole type of collective state rather than a single-particle state. The above idea has been used by Lane and Pendlebury[9] to account for the observed experimental mean life of the 3^- state of Pb^{208}. In this paper an alternative approach (i.e., the use of the idea of configuration mixing) is made which accounts for the same discrepancy as well as having some additional advantages discussed below.

As mentioned above, one can conceive of two ways of describing the 3^- state. One is to use the idea of the surface vibration model of Bohr and Mottelson[10] and the other is to use the idea of configuration mixing.[11–13] If the first one is used, the values of the parameters which are needed to describe the collective motion are fixed by the experimental values of the energy level of the excited state and transition probability of the γ-ray from the excited to the ground state. Then the cross section for the excitation of this state by direct interactions can be calculated using methods which have been considered by several authors.[14–17] In such a case, however, the strength of the interaction of the incident proton and the collective motion is not easily connected with g', the strength of interaction of the incident

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proton and the target nucleons for excitation of the single-particle levels. If the method of configuration mixing is used, the above difficulty disappears. It is usually, however, rather difficult to obtain a wave function in this way which gives a big enhancement of the electromagnetic transition probability. In Sec. II it will be shown that such a wave function can be obtained by introducing an octupole-octupole force,[18] which may be considered a natural extension of Elliott's quadrupole-quadrupole force.[19–21] It is then found that all the configurations mixed by this force add up constructively to the transition amplitude giving a large transition probability which can account for the experimental lifetime of the 3^- state of Pb^{208}.

Further in Secs. III and IV we calculate the inelastic scattering cross section of 23-MeV protons for energy levels of 0.57 and 0.90 MeV for Pb^{207} corresponding to the transitions $(p_{1/2})^{-1}\rightarrow(f_{5/2})^{-1}$ and $(p_{1/2})^{-1}\rightarrow(p_{3/2})^{-1}$, respectively, by using the direct interaction theory. The inelastic scattering cross section for the anomalous peak at 2.6 MeV (3^- state of Pb^{208}) is also calculated using a nuclear wave function with configuration mixing. In our calculations we take the interaction between the incident proton and the struck nucleon to be a three-dimensional delta function, an assumption which permits important calculational simplifications. The effect of the nucleus upon the incident proton is included through the use of the method of distorted waves in which the empirically known nuclear potential which describes the elastic scattering is used. Finally, a brief discussion of our results are given in Sec. V.

II. THE LIFETIME OF THE FIRST EXCITED 3^- STATE OF Pb^{208}

As has been discussed in the introduction, the first excited 3^- state of Pb^{208} is a collective state rather than a single-particle state. In terms of a shell model the 3^- state of Pb^{208} is usually interpreted to arise primarily from proton excitation $(d_{3/2})^4\rightarrow(d_{3/2})^{-1}h_{9/2}$. However, the theoretical value of the mean life calculated using the Weisskopf single-particle formula for this transition is larger than the experimental value by at least a factor of ten. In this section it will be shown that the above discrepancy can be removed if we introduce the configuration mixing.

The highest orbitals which are completely filled with protons in the ground configuration of Pb isotopes are $s_{1/2}$, $d_{3/2}$, and $d_{5/2}$, while the lowest unoccupied proton orbitals in the same configuration are $h_{9/2}$, $f_{7/2}$, and $f_{5/2}$. As this ground configuration corresponds to the completely filled major shell, it is clear that only a 0^+ state is possible. On the other hand, each of the above filled orbitals has even parity while the unfilled one has odd parity. Therefore, if a proton from any one of the above filled orbitals is excited to any one of the above unoccupied orbitals, then any state constructed from this new configuration has odd parity. Thus the 3^- state in question is a linear combination of all the 3^- substates constructed from the above sort of configurations.

Clearly the amplitudes (including sign) of a mixture of various substates in the 3^- (collective) state is determined if some particular interaction is assumed which has nonvanishing matrix elements between these substates. It is useful for our purpose to consider an octupole-octupole type of interaction which is analogous to the quadrupole-quadrupole type of interaction considered by other authors.[19–21] We choose the interaction of the following separable form:

$$V_{0-0}=c\sum_{i,j} r_i^3 r_j^3 Y_{3m}^*(\theta_i,\phi_i)Y_{3m}(\theta_j,\phi_j). \qquad (1)$$

If c is taken to be negative, then the assumed force is attractive.

For simplicity, we first assume that among the above substates there exists a dominant one, say $[(l_1j_1)^{-1}(l_3j_3)]_{3M}$, and the 3^- state under consideration is obtained by adding some substates to this dominant one by a first-order perturbation calculation, V_{0-0} being the perturbing interaction. Then it is easy to see that the wave function of the 3^- state in this approximation can be given by

$$\begin{aligned}|3^-\rangle &= (1/\sqrt{N})\Big(|[(l_1j_1)^{-1}(l_3j_3)]_{3M}\rangle \\ &\quad+\textstyle\sum_{2,4}[1/(E_{13}-E_{24})]\langle[(l_2j_2)^{-1}(l_4j_4)]_{3M}|V_{0-0} \\ &\quad\times|[(l_1j_1)^{-1}(l_3j_3)]_{3M}\rangle|[(l_2j_2)^{-1}(l_4j_4)]_{3M}\rangle\Big) \\ &=\frac{1}{\sqrt{N}}\Bigg(|[(l_1j_1)^{-1}(l_3j_3)]_{3M}\rangle \\ &\quad+\sum_{2,4}\frac{|c|}{|E_{13}-E_{24}|}(-)^{j_1+j_2}\langle j_1\|Y_3\|j_3\rangle \\ &\quad\times\langle j_2\|Y_3\|j_4\rangle|[(l_2j_2)^{-1}(l_4j_4)]_{3M}\rangle\Bigg). \qquad (2)\end{aligned}$$

In (2) the ket vectors are the wave functions of the above substates; E_{ik} is the energy difference between the configuration $[(l_ij_i)^{-1}(l_kj_k)]_{3M}$ and the ground configuration; sum over 2 and 4 means the summation is to be performed over all possible substates in which at least one of the two inequalities $(l_1j_1)\neq(l_2j_2)$ and $(l_3j_3)\neq(l_4j_4)$ holds, and from the definition of the dominant configuration it can be considered that $E_{13}-E_{24}<0$. The double-barred quantity $\langle j_1\|Y_3\|j_3\rangle$ is defined by the relation

$$\langle j_1m_1|r^3Y_{3m}|j_3m_3\rangle =\langle j_1\|Y_3\|j_3\rangle(-)^{m_1+m}(j_1-m_1j_3m_3|3-m), \qquad (3)$$

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where

$$\langle j_1\|Y_3\|j_3\rangle = -[1/(7\times4\pi)^{1/2}]\langle r^3\rangle_{l_1l_3} i^{l_1-l_3} Z(l_1j_1l_3j_3;\tfrac{1}{2}\,3),$$

$(j_1-m_1j_3m_3|3-m)$ being the Clebsch-Gordan coefficient and Z being the Z coefficient defined by Biedenharn *et al.*[22] It should be noted that $\langle j_1\|Y_3\|j_3\rangle$ is a pure imaginary quantity.

The matrix element of the $E3$-transition operator between the state (2) and the ground state $|0^+\rangle$, which is assumed to consist purely of the ground configuration, can be written as $\langle 3^-|c'\sum_i r_i^3 Y_{3m}(\theta_i\phi_i)|0^+\rangle$, where the summation is over all the existent protons and c' is an appropriate constant. (It is perhaps worthwhile to note that we assume neutrons to be without charge; consequently neutron configurations are neglected.) The evaluation of this matrix element is straightforward and the result is

$$\begin{aligned}\langle 3^-|\textstyle\sum_i c' r_i^3 Y_{3m}(\theta_i,\phi_i)|0^+\rangle \\ = (c'/\sqrt{N})(-)^{1+j_1}\langle j_1\|Y_3\|j_3\rangle \\ \times(1+\textstyle\sum_{2,4}[|c|/|E_{13}-E_{24}|]|\langle j_2\|Y_3\|j_4\rangle|^2).\end{aligned}\quad(4)$$

It is worth noting that the contributions from all the mixed substates to this matrix element are additive, which guarantees that the transition probability is certainly enhanced compared to the case in which the 3^- state is described simply by the dominant configuration.

In the actual case, the ratio $|c|/|E_{13}-E_{24}|$ might not be small compared to unity. In such a case the above perturbation calculation is not accurate enough and it is necessary to solve a secular equation. Even in such a case, however, it is expected that the relative phase of the mixture of each substate in the ground-state wave function will be the same as it is in (2). It is further expected that the magnitude of the amplitude of each of the mixed substates will not differ very much from each other. Therefore, and particularly as we do not have any reliable knowledge of the quantities $(E_{13}-E_{24})$, the best we can do is to replace (2) by

$$|3^-\rangle = \frac{1}{(N_1)^{1/2}}\sum_{1,3}(-)^{1+j_1}\frac{\langle j_1\|Y_3\|j_3\rangle}{|\langle j_1\|Y_3\|j_3\rangle|} \times|[(l_1j_1)^{-1}(l_3j_3)]_{3M}\rangle,\quad(5)$$

where now the summation over 1 and 3 runs over all the substates. By using (5) the matrix element (4) is reduced to the following simple form:

$$\langle 3^-|\textstyle\sum_i c' r_i^3 Y_{3m}(\theta_i\phi_i)|0^+\rangle = [c'/(N_1)^{1/2}]\textstyle\sum_{1,3}|\langle j_1\|Y_3\|j_3\rangle|,\quad(6)$$

where the additivity of all the contributions is naturally retrained. In (5) and (6) the normalization constant N_1 is just equal to the number of the mixed substates.

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As the second order approximation we next consider the mixture of some higher order configurations with the ground state. Because of the above-mentioned considerations on the parity of the orbitals involved, it is clear that the configurations which can be mixed to the ground state should have two protons excited from the filled orbitals to the unoccupied ones. These states could be written, with obvious notation, as $|\{[(l_1j_1)^{-1}(l_3j_3)]_3[(l_2j_2)^{-1}(l_4j_4)]_3\}_{0^+}\rangle$. Now the mixture of these new substates to the dominant substate of the ground state (i.e., the 0^+ state which results from the ground configuration), could be obtained by the simple perturbational calculation because the energy difference $|E_{13}+E_{24}|$ between these substates and the dominant substate could be much bigger than (c). Thus the new wave function of the ground state can be written as

$$\begin{aligned}|0^+\rangle = \frac{1}{(N_2)^{1/2}}\Bigg(&|\text{filled shells}\rangle_{0^+} \\ &+\sum_{1,2,3,4}\frac{7\times7^{1/2}|c|}{|E_{13}+E_{24}|}(-)^{1+j_1+j_2} \\ &\times\langle j_1\|Y_3\|j_3\rangle\langle j_2\|Y_3\|j_4\rangle[1-(\text{exch})]| \\ &\times|\{[(l_1j_1)^{-1}(l_3j_3)]_3[(l_2j_2)^{-1}(l_4j_4)]_3\}_{0^+}\rangle\Bigg),\end{aligned}\quad(7)$$

where the exchange term in the square brackets is given by

$$\begin{aligned}(\text{exch}) = &[\langle j_1\|Y_3\|j_4\rangle\langle j_2\|Y_3\|j_3\rangle] \\ &\times[\langle\langle j_1\|Y_3\|j_3\rangle\langle j_2\|Y_3\|j_4\rangle]^{-1} \\ &\times(-)^{j_1+j_2+j_3+j_4}W(j_1j_4j_3j_2;33).\end{aligned}\quad(8)$$

Although the sign of (exch) is not unique, its magnitude is at most of the order of 0.1 and thus the sign of the first square bracket in (7) is positive definite.

The matrix element of the operator $\sum_i r_i^3 Y_{3m}(\theta_i\phi_i)$ between states (7) and (5) can be easily calculated and the result is

$$\begin{aligned}&\langle 3^-|\textstyle\sum_i c' r_i^3 Y_{3m}(\theta_i\phi_i)|0^+\rangle \\ &= [c'/(N_1N_2)^{1/2}]\Bigg[\textstyle\sum_{1,3}|\langle j_1\|Y_3\|j_3\rangle| \\ &\quad+\sum_{1,2,3,4}\frac{7|c|}{2|E_{13}+E_{24}|}|\langle j_1\|Y_3\|j_3\rangle|^2|\langle j_2\|Y_3\|j_4\rangle|^2 \\ &\quad\times\left(\frac{1}{|\langle j_1\|Y_3\|j_3\rangle|}+\frac{1}{|\langle j_2\|Y_3\|j_4\rangle|}\right)[1-(\text{exch})]\Bigg].\end{aligned}\quad(9)$$

It is clearly seen in (9) that all the calculated matrix elements add coherently.

It would now be of interest to estimate the order of magnitude of the transition probability derived from the matrix element (9). As we do not know the precise

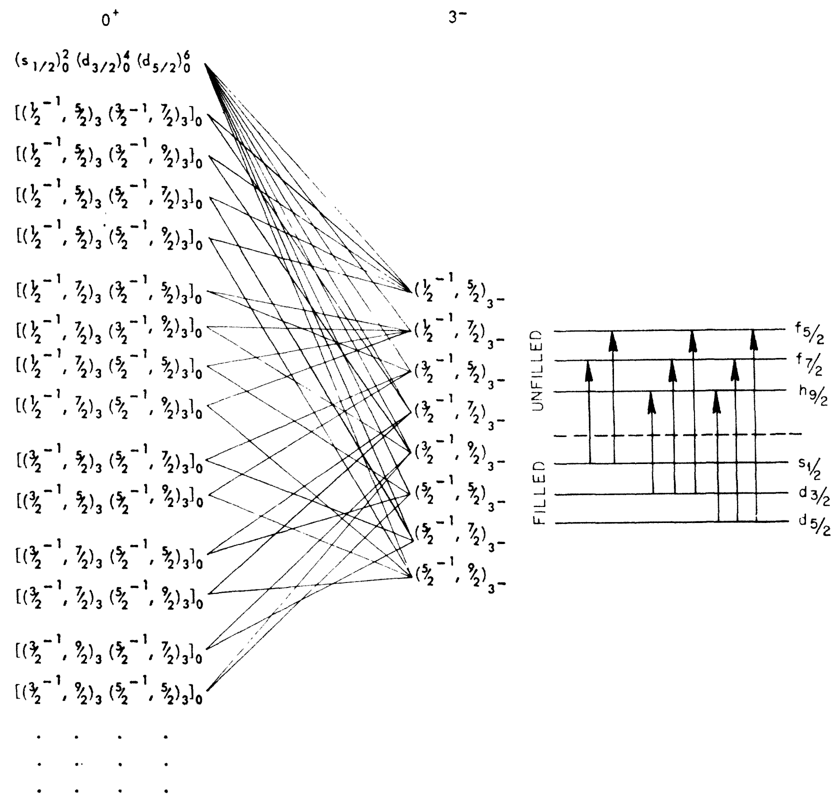

FIG. 1. Some of the states mixed to the ground configuration, and the connecting matrix elements between them and the $|3^-\rangle$ state.

magnitude of the quantity c or of the energy differences of different orbitals, a very precise evaluation is impossible to do and the following simplified version will suffice for our purpose. First, the magnitude of the reduced matrix element $\langle j_1\|Y_3\|j_3\rangle$ fluctuates from some particular values of j_1 and j_3 to the others within a factor of three, but its average value is quite close to unity, the similar reduced matrix element corresponding to Weisskopf's simplified model being taken as the unit. Secondly, the amplitudes of the terms which appear under the summation symbol in (7) may all be of the same order of magnitude and thus (7) may be approximated by

$$|0^+\rangle=[1/(N_2)^{1/2}](|\text{filled shells}\rangle_{0^+}+\eta\sum_{1,2,3,4}(\pm)\times|\{[(l_1j_1)^{-1}(l_3j_3)]_3[(l_2j_2)^{-1}(l_4j_4)]_3\}_{0^+}\rangle),\quad(10)$$

the sign factor being taken as it is in (7). Here η is a constant which will be much smaller than unity.

If we restrict our consideration to those configurations which were mentioned in the beginning of this section, it can be shown that the normalization constants which appear in (9) are given by $N_1=8$ and $N_2=1+36\eta^2$, and thus the square of (9), again measured by taking Weisskopf's value as the unit, is given by $(8+64\eta)^2/8(1+36\eta^2)$. (See Fig. 1.) This is already equal to eight for $\eta=0$ and increases rapidly with the increasing η. Therefore, at least qualitatively, it will be quite easy to explain in this way the experimental value of the transition probability which is known to be at least ten times larger than the Weisskopf value.

It is clear that the model so far considered is quite crude. It would be certainly necessary to take into account the effect of some short-range interaction in calculating the configuration mixing. For example, if we consider as a short-range interaction, the so-called pairing interaction,[23,24] configurations like $\{[(l_1j_1)^{-2}]_0\times[(l_3j_3)^2]_0\}$ will be also mixed into (7) due to this interaction and the relative amplitude of the above configurations will be affected. Further, the matrix elements of the $E3$ transition related to the newly considered configurations might be out of phase, in which case the above-obtained transition probability would be decreased. The value discussed above is, however, quite large, and furthermore if necessary it is possible to increase the magnitude by adding higher configurations such as $h_{11/2}$, $i_{13/2}$ which have been neglected in the above.

The actual magnitude may be determined if the relative strength of the octupole-octupole and the pairing interaction (and also the energy difference between different orbitals) are known. As they are not known, it would not be of much use to pursue the discussion in further detail here. It is possible, nevertheless, to conclude that we know at least in principle the way to explain the large transition probability by describing the related states exclusively in terms of the

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[24] S. T. Belyaev, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 31, No. 11 (1959).

individual particle model, (i.e., without taking into account the collective model). This idea will be utilized in Sec. IV in calculating the cross section of the excitation of the 3^- state of Pb^{208}.

III. THE CALCULATION OF THE INELASTIC CROSS SECTION FOR Pb^{207}

In this calculation we shall fix our attention on a particular example which can be easily generalized for other cases as well. It would not be a very poor approximation for our purpose to consider that the ground, first, and second excited states of Pb^{207} have, respectively, the following pure neutron configurations: $(p_{1/2})^{-1}$, $(f_{5/2})^{-1}$, and $(p_{3/2})^{-1}$ while all the 82 protons form the closed major shells. Therefore, in this model the transition from the ground state to each of these excited states is simply a transition of a single neutron hole to another, i.e., $[(p_{1/2})^{-1}\rightarrow(lj)^{-1}]$. In the shell-model calculation this process turns out to be equivalent to a particle to particle transition. For derivation of the cross section in the distorted-wave approximation, we shall follow the theory of Lamarsh and Feshbach.[25] Let us consider the scattering of an incident particle from an initial nuclear state $|i\rangle$ to a final state $|f\rangle$ with the incident particle wave function $\phi_0(\mathbf{r})$ with wave number K and energy E and inelastically scattered wave function $\phi_1(\mathbf{r})$ with energy E' and wave number K' in the center-of-mass coordinate system. Then to a first approximation $\phi_0(\mathbf{r})$ and $\phi_1(\mathbf{r})$ will satisfy the following coupled equations:

$$[-(\hbar^2/2\mu)\nabla^2+U(r)-E]\phi_0(\mathbf{r})=0, \tag{11}$$

$$[-(h^2/2\mu)\nabla^2+U(r)-E']\phi_1(\mathbf{r})=-\langle f|H'|i\rangle\phi_0(\mathbf{r}), \tag{12}$$

where $U(r)$ is the optical-model potential which produces the observed elastic scattering from the same nucleus at the same energy and H' is the effective interaction; in other words, it is the remainder of the total interaction between the incident particle and the target nucleus after the average interaction with the target nucleus "U" has been subtracted. Here the wave function $\phi_0(\mathbf{r})$ represents the sum of the incident and the elastically scattered outgoing wave and the wave function $\phi_1(\mathbf{r})$ represents the inelastically scattered outgoing wave only. Therefore, $\phi_0(\mathbf{r})$ and $\phi_1(\mathbf{r})$ have the asymptotic forms

$$\phi_0(\mathbf{r}) \underset{r\to\infty}{\longrightarrow} e^{iKz}+(1/r)f_0(\theta,\phi)e^{iKr}, \tag{13}$$

$$\phi_1(\mathbf{r}) \underset{r\to\alpha}{\longrightarrow} -(e^{iK'r}/r)f_1(\theta,\phi), \tag{14}$$

where $f_0(\theta,\phi)$ and $f_1(\theta,\phi)$ are the elastically and inelastically scattering amplitudes, respectively. Therefore, the differential inelastic scattering cross section is given by

$$d\sigma(\theta)=(K'/K)\sum_{\mathrm{av}}|f_1(\theta\phi)|^2, \tag{15}$$

where the symbol $\sum_{\mathrm{av}}$ is used to represent a summation over final spins and an average over initial spins. Thus our problem is reduced to obtain the solution of (12) which has the asymptotic form of (14). This equation can be solved by the Green's function method (Mott and Massey[26]) and for large r the wave function ϕ_1 has the asymptotic form

$$\phi_1(r) \underset{r\to\infty}{\longrightarrow} -\frac{e^{iK'r}}{r}\left(\frac{\mu}{2\pi\hbar^2}\right) \times\int\langle f|H'|i\rangle\phi_0(r'\theta')K(\mathbf{r},\mathbf{r}')d\tau', \tag{16}$$

where

$$K(\mathbf{r},\mathbf{r}')=\sum_L (2L+1)e^{i\eta_L}e^{-\frac{1}{2}iL\pi}G_L(r')P_L(\cos\Theta), \tag{17}$$

Θ being the angle between the vectors $\mathbf{r}$ and $\mathbf{r}'$ and $G_L(r)$ is the solution of the homogeneous wave equation

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dG_L}{dr}\right)+\left[K'-\frac{2\mu}{\hbar^2}U(r)-\frac{L(L+1)}{r^2}\right]G_L(r)=0. \tag{18}$$

Now, comparing equation (16) with that of (14), we obtain

$$f_1(\theta,\phi)=\left(\frac{\mu}{2\pi\hbar^2}\right)\int\langle f|H'|i\rangle\phi_0(r'\theta')K(\mathbf{r},\mathbf{r}')d\tau'.$$

Therefore, from (15) we have the differential inelastic scattering cross section

$$d\sigma(\theta)=\frac{K'}{K}\left(\frac{\mu}{2\pi\hbar^2}\right)^2\sum_{\mathrm{av}}\left|\int\langle f|H'|i\rangle\phi_0(r'\theta')K(\mathbf{r},\mathbf{r}')d\tau'\right|^2. \tag{19}$$

Evaluation of the Matrix Element of the Interaction Operator H'

For the effective interaction H' between the nucleons of the target nucleus and the incident proton we take a three-dimensional delta function. Let

$$H'=g'\sum_n\delta(\mathbf{r}_n-\mathbf{r}), \tag{20}$$

where $\mathbf{r}_n$ is the position vector for the nucleon undergoing transition from one state to another and $\mathbf{r}$ is that of the incident proton. Expanding the δ function we have

$$H'=g'\sum_{nqm_q}[\delta(r_n-r)/r^2]Y_{qm_q}(\theta_n\phi_n)Y_{qm_q}{}^*(\theta\phi), \tag{21}$$

where Y's are the normalized spherical harmonics.

Now suppose we are considering the excitation from an initial nuclear state $|J_iM_i\rangle$ to a final state $|J_fM_f\rangle$ which is due to a single-particle transition and the remaining core of the nucleus has zero spin. Then by means of Racah's algebra, we can easily evaluate the

[25] J. R. Lamarsh and H. Feshbach, Phys. Rev. **104**, 1633 (1956).

[26] N. F. Mott and H. S. Massey, *The Theory of Atomic Collisions* (Oxford University Press, New York, 1949), 2nd ed.

matrix element and the final result is given by

$$\langle f|H'|i\rangle = g'\frac{i^{l_f-l_i+1}}{(4\pi)^{1/2}}(-)^{M_f-J_f-J_i}[(2J_i+1)(2J_f+1)]^{1/2}$$
$$\times\sum_q (i)^q\frac{1}{(2q+1)^{1/2}}(J_f-M_fJ_iM_i/qM_i-M_f)$$
$$\times W(J_fJ_fJ_iJ_i;0q)Z(l_fJ_fl_iJ_i;\tfrac{1}{2}q)$$
$$\times Y_q^{M_i-M_f}(\theta\phi)R_{n_il_i}(r)R_{n_fl_f}(r), \quad (22)$$

where $R_{n_il_i}(r)$ and $R_{n_fl_f}(r)$ are the radial functions for the orbital nucleon undergoing transition in the ground and excited states, respectively.

Now let us consider the function ϕ_0 and G, the solution of the homogeneous wave equations (11) and (18). For the effective average potential $U(r)$ we assume that it is spherical symmetric, and we write it into two parts

$$U(r)=U_s(r)+U_c(r), \quad (23)$$

where $U_s(r)$ is the optical potential without Coulomb interaction and is given by

$$U_s=-(V_0+iW)1/\{1+\exp[(r-R)/a]\}, \quad (24)$$

and $U_c(r)$ is the Coulomb interaction potential. U_s and U_c satisfy the following asymptotic conditions:

$$rU_s\to 0 \quad \text{for} \quad r\to 0,$$

and

$$rU_c\to 2\eta E/K \quad \text{for} \quad r>R,$$

and inside the nucleus,

$$U_c=(ZZ'e^2/2R)[3-(r/R)^2] \quad \text{for} \quad r<R,$$

where $\eta=ZZ'e^2\mu/\hbar^2K$; Ze and $Z'e$ are the electric charges for the nucleus and the incident particle, respectively.

Expanding

$$\phi_0=\sum_{L'}[\chi_{L'}(r)/r]P_{L'}(\cos\theta), \quad (25)$$

and

$$P_L(\cos\Theta)=[4\pi/(2L+1)]\sum_m Y_{Lm}(\theta\varphi)Y_{Lm}^*(\theta'\varphi'), \quad (26)$$

and substituting f_L/r for G_L, then

$$K(\mathbf{r},\mathbf{r}')=\sum_L\sum_m 4\pi e^{i\eta_L}e^{-\frac{1}{2}iL\pi}(f_L/r)\times Y_{Lm}(\theta,\phi)Y_{Lm}^*(\theta',\phi'). \quad (27)$$

Now making the substitutions in Eq. (19) for ϕ_0 from Eq. (25), $K(\mathbf{r},\mathbf{r}')$ from (27), $\langle f|H'|i\rangle$ from (22); integrating over the angular coordinates, averaging over initial states and summing over final states, we get the differential cross section after certain simplifications:

$$d\sigma(\theta)=g'^2\left(\frac{\mu}{2\pi\hbar^2}\right)^2\frac{K'}{K}\frac{1}{2J_i+1}$$
$$\times\Big|\sum_{LL'L_1L_1'qn}(-)^q(i)^{L_1'-L'+L_1-L-n}$$
$$\times(L0q0/L'0)(L_10q0/L_1'0)(L_1'0L'0/n0)$$
$$\times Z(L_1L_1'LL';qn)Z^2(l_fJ_fl_iJ_i;\tfrac{1}{2}q)$$
$$\times H_{L'L}^{(l_il_f)}H_{L_1'L_1}^{(l_il_f)*}P_n(\cos\theta)\Big|, \quad (28)$$

where $H_{L'L}^{(l_il_f)}$ is defined by

$$H_{L'L}^{(l_il_f)}=(-)^{L'}(i)^{L'+1}\frac{1}{(2L+1)^{1/2}(2L'+1)}e^{i\eta L'}\left(\frac{B_{L'}}{N_{L'}}\right)\left(\frac{B_L}{N_L}\right)$$
$$\times\int\chi_L(r)f_{L'}(r)R_{n_il_i}(r)R_{n_fl_f}(r)dr,$$

where (B_L/N_L) and $(B_{L'}/N_{L'})$ are the normalization constants and $\chi_L(r)$ and $f_{L'}(r)$ satisfy the following differential equation:

$$\frac{d^2f_{L'}(r)}{dr^2}+\frac{2\mu}{\hbar^2}\left[E'-U_s-U_c-\frac{L'(L'+1)}{r^2}\frac{\hbar^2}{2\mu}\right]f_{L'}(r)=0, \quad (29)$$

$$\frac{d^2\chi_L(r)}{dr^2}+\frac{2\mu}{\hbar^2}\left[E-U_s-U_c-\frac{L(L+1)}{r^2}\frac{\hbar^2}{2\mu}\right]\chi_L(r)=0. \quad (30)$$

Now Eq. (28) is completely defined and using this expression we can calculate the differential cross section $d\sigma(\theta)$.

IV. THE INELASTIC SCATTERING CROSS SECTION FOR ANOMALOUS PEAK AT 2.6 MeV OF Pb208 CORRESPONDING TO ITS FIRST EXCITED 3⁻ STATE

In the previous section we have seen that the life time of the first excited 3^- state can be approximately accounted for by using wave functions with configurating mixing. Here we use the *same wave functions* to calculate the inelastic cross section for this state. As in Sec. III we assume here again that the interaction between the incident proton and the struck nucleon is a three-dimensional delta function. Let us write the wave functions for the ground state $|0^+\rangle$ and the first excited state $|3^-\rangle$ as follows, in obvious notation

$$|0^+\rangle=\frac{1}{(N!/n_1!n_2!\cdots)^{1/2}}\sum_P(\pm)\psi(j_1^{n_1})_0\psi(j_2^{n_2})_0\cdots$$
$$=\frac{1}{(N!/n_1!n_2!\cdots)^{1/2}}\sum_P(\pm)(1\cdots n_1-1,n_1)_0$$
$$\times(1\cdots n_2-1,n_2)_0(\cdots)_0\cdots, \quad (31)$$

where $n_1, n_2\cdots$ are the number of equivalent nucleons forming closed shells with spins $j_1, j_2, \cdots$, respectively, and N is the total number of nucleons (i.e., $n_1+n_2+n_3+\cdots=N$). The wave function for $|3^-\rangle$ state corresponding to Eq. (5) of Sec. II where one particle has made a transition from a closed shell (l_1j_1) to an empty shell (l_3j_3) can be written

$$|3^-\rangle=\sum_{j_1j_3}a_{j_1j_3}|[(l_1j_1)^{-1}(l_3j_3)]_{3M}\rangle$$
$$=\frac{1}{(N!/(n_1-1)!n_2!\cdots)^{1/2}}\sum_P(\pm)a_{j_1j_3}\times[\psi(j_1^{n_1-1})_{j_1}\psi(j_3^1)_{j_3}]_{3M}\psi(j_2^{n_2})_0\cdots$$
$$=\frac{1}{(N!/(n_1-1)!n_2!\cdots)^{1/2}}\sum_{Pm_1m_3}(\pm)a_{j_1j_3}\times(1\cdots n_1-1)_{j_1m_1}(j_3^1)_{j_3m_3}(1\cdots n_2)_0\cdots\times(j_1m_1j_3m_3/3M),\quad(32)$$

where $a_{j_1j_3}$ is the amplitude of the state

$$|[(l_1j_1)^{-1}(l_3j_3)]_{3M}\rangle,$$

and its phase is given by Eq. (5) and its magnitude is taken to be independent of j's values; $(j_1m_1j_3m_3|3M)$ is the Clebsch-Gordan coefficient.

From Eqs. (21), (31), and (32), the calculation of the matrix element gives the result

$$\langle 3^-|H'|0^+\rangle=g'\sum_{j_1j_3}|a_{j_1j_3}Z(l_1j_1l_3j_3;\tfrac{1}{2}3)|\frac{1}{(7\times4\pi)^{1/2}}(-)^MR_{n_1l_1}(r)\times R_{n_3l_3}(r)Y_{3M}(\theta\varphi),\quad(33)$$

where $R_{n_1l_1}(r)$ and $R_{n_3l_3}(r)$ corresponding to the initial and final-state radial wave functions for the nucleon undergoing transition.

Substituting for the matrix element $\langle 3^-|H'|0^+\rangle$ from Eq. (33) in Eq. (19) and following through all the steps for deriving Eq. (28) of Sec. III, we obtain for the differential cross section

$$d\sigma(\theta)=g'^2\left(\frac{\mu}{2\pi\hbar^2}\right)^2\frac{K'}{K}\Big(\sum_{j_1j_3}|a_{j_1j_3}Z(l_1j_1l_3j_3;\tfrac{1}{2}3)|\Big)^2\times\Big|\sum_{LL'L_1L_1'n}(i)^{L_1+L_1'-L-L'-n}\times(L030/L'0)(L_1030/L_1'0)(L_1'0L'0/n0)\times Z(L_1L_1'LL';3n)H_{L'L}{}^*H_{L_1'L_1}P_n(\cos\theta)\Big|.\quad(34)$$

In obtaining the above equation we have taken the radial integrals $H_{L'L}{}^{(l_1l_3)}\cdots$ defined in Sec. III to be independent of l values, and have omitted the superscripts on $H_{L'L}{}^{(l_1l_3)}\cdots$.

In the second-order approximation we introduce the configuration mixing to the ground state as it was done in Sec. II, i.e., we take Eq. (7) of Sec. II for the wave function of the ground state $|0^+\rangle$ instead of Eq. (31) of this section. Then, with the approximations used in Sec. II and in view of the detailed consideration obtained therein as well as from the work of Pinkston and Satchler,[27] we infer that

$$d\sigma(\theta)=T(E3)g'^2\left(\frac{\mu}{2\pi\hbar^2}\right)^2\frac{K'}{K}\langle|Z(l_1j_1l_3j_3;\tfrac{1}{2}3)|\rangle_{\rm av}{}^2\times\Big|\sum_{LL'L_1L_1'n}(i)^{L_1+L_1'-L-L'-n}\times(L030/L'0)(L_1030/L_1'0)(L_1'0L'0/n0)\times Z(L_1L_1'LL';3n)H_{L'L}{}^*H_{L_1'L_1}P_n(\cos\theta)\Big|,$$

where $\langle|Z(l_1j_1l_3j_3;\frac{1}{2}3)|\rangle_{\rm av}$ is the average of the absolute values of $Z(l_1j_1l_3j_3;\frac{1}{2}3)$ of all the mixed configurations and $T(E3)$ is the ratio of the transition probability of the 3^- state using nuclear wave functions with configuration mixing to the Weisskopf's on the single-particle model. We use this expression to calculate the inelastic cross section for the 3^- state of Pb^{208}. The value of $T(E3)$ may be taken ten to twenty-five as estimated from the expression $(8+64\eta)^2/8(1+36\eta^2)$ [here η is defined by Eq. (10)].

V. RESULTS AND DISCUSSIONS

In this section we give the results of some calculations of the inelastic cross section by the direct interaction process and compare them with the experimental ones. A brief discussion of our present calculations is also given. The calculations have been performed by taking for U_s a Saxon well of the form[28]

$$U_s=-(V_0+iW)/\{1+\exp[(r-R)/a]\}$$

where $V_0=44.3$ MeV, $W=11.8$ MeV, $a=0.5\times10^{-13}$ cm, $R=1.33A^{1/3}\times10^{-13}$ cm and for the Coulomb interaction U_c the following form:

$$U_c=(Z_1Z_2e^2/2R)[3-(r/R)^2]\quad\text{for}\quad r<R,$$

where Z_1 and Z_2 are the number of electric charges for the nucleus and the incident particle, respectively. For the radial wave function of the nucleon bound inside the nucleus we have taken the harmonic oscillator function given by

$$R_{nl}(r)\sim\exp(-r^2/2b^2)r^lL_{n+l+\frac{1}{2}}{}^{l+\frac{1}{2}}(r^2/b^2),$$

where $L_q{}^p(r)$ is an associated Laguerre polynomial, $b^2=\hbar/MW=2.33\times10^{-13}$ cm.[29] Thus in our present calculation the effect of the nucleus upon the incident proton has been included through the use of the method

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of distorted waves as discussed in the previous sections. The above-mentioned nuclear parameters which give the best fit to the elastic scattering data have been used. The interaction between the incident proton and the target nucleus has been considered throughout the entire volume of the nucleus. The absolute inelastic scattering cross sections of 23 MeV protons at 90° for energy levels of 0.57 and 0.90 MeV for Pb^{207} corresponding to the transitions $(p_{1/2})^{-1} \rightarrow (f_{5/2})^{-1}$ and $(p_{1/2})^{-1} \rightarrow (p_{3/2})^{-1}$ are obtained $\approx (g'\mu/2\pi\hbar^2)^2 \times 3 \times 10^{-3}$ and $\approx (g'\mu/2\pi\hbar^2)^2 \times 2.5 \times 10^{-3}$, respectively.

The differential inelastic scattering cross section of 23-MeV protons at 90° for the 2.6-MeV $|3^-\rangle$ state of Pb^{208} is found to be $\approx T(E3)(g'\mu/2\pi\hbar^2)^2 \times 1.5 \times 10^{-3}$. Now if we suppose that the anomalous peaks at 2.6 MeV for Pb^{206} and Pb^{207} are also due to the superpositions of many single-particle transitions as in Pb^{208} then the cross sections for them will also be given by the same value. We have estimated ten to twenty-five as a reasonable value for $T(E3)$ in Sec. IV and if we take $(g'\mu/2\pi\hbar^2)^2 \approx 100$ mb/sr as estimated by Lamarsh and Feshbach,[25] the cross section for the anomalous peak is found to be 1.5–3.8 mb/sr which is very reasonable compared to the value (≈ 2.5 mb/sr) measured by Cohen and Mosko.[3]

With the same value for $(g'\mu/2\pi\hbar^2)^2$, the cross sections for 0.57- and 0.90-MeV levels of Pb^{207} are found to be 0.3 mb/sr and 0.25 mb/sr, respectively. The ratios between the cross sections for these levels to the anomalous peak at 2.6 MeV are between 0.20 to 0.08 for 0.57-MeV level and between 0.16 to 0.07 for 0.90-MeV level. These are to be compared with the experimental ratios which are approximately 0.1 and slightly smaller than 0.1, respectively, as obtained from Cohen and Rubin's measurements[4] (if the background which exists in their experiment is taken into account). Thus we see that the agreement between the experimental and theoretical cross sections for the anomalous peaks at 2.6 MeV of Pb^{206}, Pb^{207}, and Pb^{208} as well as for the 0.57- and 0.90-MeV levels of Pb^{207} is fairly satisfactory. These results at least in principle show how one can understand the 3^- collective state and also the anomalous peaks in (p,p') reaction observed at 2.6 MeV in Pb isotopes in terms of the single-particle picture. This description can also be applied to the other first excited 3^- state.[30]

In conclusion, we hope that our present analysis may serve as a useful guide for obtaining more realistic configuration—mixed nuclear wave function as a result of residual interaction. It is believed that the above approach can also serve as a guide for more refined inelastic scattering calculations where it might be necessary to take into account a finite range of interaction between the incident proton and the struck nucleon; it might even be necessary to vary the Saxon well parameters as was done by Levinson and Banerjee.[31] Finally it should be noted that our analysis has been limited to a few cases and it would be interesting to examine more cases in the above spirit. This will certainly clarify some of the difficulties which we have in understanding the collective states in terms of the independent-particle model.

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